AP Calculus AB
Formulas & Justifications

Justifying that a function is continuous at a point:

f is continuous at c iff:

1. $f(c)$ is defined
2. $\lim_{x \to c} f(x)$ exists
3. $f(c) = \lim_{x \to c} f(x)$

Intermediate Value Theorem:

If f is continuous on [a, b] and k is any number between $f(a)$ and $f(b)$, then there is at least one number c between a and b such that $f(c) = k$.

Definition of the Derivative:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$ (Alternate form for a derivative at a given value.)

Justifying that a derivative exists at a point, c:

Show algebraically that $\lim_{x \to c^-} f'(x) = \lim_{x \to c^+} f'(x)$.

Average Rate of Change of f on [a, b]:

a.r.c. = $\frac{f(b) - f(a)}{b - a}$ (algebra slope of $\frac{\Delta y}{\Delta x}$) (slope of secant line)

Instantaneous Rate of Change of f at a:

$f'(a)$ (derivative at the given value) (slope of tangent line)

Power Rule:

$$\frac{d}{dx}\left[x^n\right] = nx^{x-1}$$

Product Rule:

$$\frac{d}{dx}\left[uv\right] = uv' + vu'$$

Quotient Rule:

$$\frac{d}{dx}\left[\frac{u}{v}\right]=\frac{vu'-uv'}{v^2}$$

Common Derivatives to Remember:

$$\frac{d}{dx}\left[\frac{1}{x}\right]=\frac{-1}{x^2} \qquad \frac{d}{dx}\left[\sqrt{x}\right]=\frac{1}{2\sqrt{x}}$$

Trig Function Derivatives:

$$\frac{d}{dx}[\sin x]=\cos x \qquad \frac{d}{dx}[\cos x]=-\sin x$$

$$\frac{d}{dx}[\tan x]=\sec^2 x \qquad \frac{d}{dx}[\cot x]=-\csc^2 x$$

$$\frac{d}{dx}[\sec x]=\sec x\tan x \qquad \frac{d}{dx}[\csc x]=-\csc x\cot x$$

Chain Rule:

$$\frac{dy}{dx}=\frac{dy}{du}\bullet\frac{du}{dx} \qquad \frac{d}{dx}[f(g(x))]=f'(g(x))\bullet g'(x)$$

Justifications for linear approximation estimates:

A linear approximation (tangent line) is an *overestimate* if the curve is concave down.

A linear approximation (tangent line) is an *underestimate* if the curve is concave up.

Justifications for Particle Motion:

Particle is moving right/up because $v(t)>0$ (positive).

Particle is moving left/down because $v(t)<0$ (negative).

Particle is speeding up (|velocity| is getting bigger) because $v(t)$ and $a(t)$ have same sign.

Particle is slowing down (|velocity| is getting smaller) because $v(t)$ and $a(t)$ have different signs.

Justifications for horizontal and vertical tangent lines:

$f(x)$ has horizontal tangents when $\frac{dy}{dx}=0$.

$f(x)$ has vertical tangents when $\frac{dy}{dx}$ is undefined.

Derivatives of Inverse Trig Functions:

$$\frac{d}{dx}[\arcsin x]=\frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}[\arccos x]=\frac{-1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}[\arctan x]=\frac{1}{1+x^2} \qquad \frac{d}{dx}[arc\cot x]=\frac{-1}{1+x^2}$$

$$\frac{d}{dx}[arc\sec x]=\frac{1}{|x|\sqrt{x^2-1}} \qquad \frac{d}{dx}[arc\csc x]=\frac{-1}{|x|\sqrt{x^2-1}}$$

Derivatives of Inverse Functions:

The derivative of an inverse function is the reciprocal of the derivative of the original function at the "matching" point.

If (a, b) is on $f(x)$, then (b, a) is on $f^{-1}(x)$ and $(f^{-1})'(b)=\frac{1}{f'(a)}$.

Derivatives of Exponential and Logarithmic Functions:

$$\frac{d}{dx}[\ln x]=\frac{1}{x},\ x>0 \qquad \frac{d}{dx}[\log_a x]=\frac{1}{x\ln a}$$

$$\frac{d}{dx}[e^x]=e^x \qquad \frac{d}{dx}[a^x]=a^x\ln a$$

Extreme Value Theorem:

If f is continuous on the closed interval [a, b], then f has both a minimum and a maximum on the closed interval [a, b].

Justification for an Absolute Extrema.

1. Find critical numbers.
2. Identify endpoints.
3. Find f(critical numbers) and f(endpoints).
4. Determine absolute max/min values by comparing the y-values. State in a sentence.

Mean Value Theorem:

If f is continuous on [a, b] and differentiable on (a, b) then there exists a number c on (a, b) such that $f'(c)=\frac{f(b)-f(a)}{b-a}$. (Calculus slope = Algebra Slope)

Rolle's Theorem:

If f is continuous on [a, b] and differentiable on (a, b) and if $f(a)=f(b)$, then there exists a number c on (a, b) such that $f'(c)=0$.

Justification for a Critical Number:

$x=c$ is a critical number because $f'(x)=0$ or $f'(x)$ is undefined.

Justification for Increasing/Decreasing Intervals:

Inc: $f(x)$ is increasing on [_____, _____] b/c $f'(x)>0$.

Dec: $f(x)$ is decreasing on [_____, _____] b/c $f'(x)<0$.

Justification for a Relative Max/Min Using 1st Derivative Test:

Local Max: A relative max value of _______ exists at x = ______ b/c $f'(c)=0$ (or und) and $f'(x)$ changes from + to -.

Local Min: A relative min value of _______ exists at x = ______ b/c $f'(c)=0$ (or und) and $f'(x)$ changes from - to +.

Justification for Relative Max/Min Using 2nd Derivative Test:

Local Max: A relative max value of _______ exists at x = _______ b/c $f'(c)=0$ (or und) and $f''(x)<0$.

Local Min: A relative min value of _______ exists at x = _______ b/c $f'(c)=0$ (or und) and $f''(x)>0$.

Justification for a Point of Inflection:

Using 2nd derivative: x = _______ is a point of inflection b/c $f''(\_\_\_\_)=0$ (or dne) AND $f''(x)$ changes sign at x = _______.

Using 1st derivative: x = _______ is a point of inflection b/c $f''(\_\_\_\_)=0$ (or dne) AND $f'(x)$ changes from inc to dec (or from dec to inc).

Justification for Concave Up/Concave Down:

Concave Up: $f(x)$ is concave up on (_____, _____) because $f''(x)>0$.

Concave Down: $f(x)$ is concave down on (_____, _____) because $f''(x)<0$

Integration Rules:

$\int \cos x dx = \sin x + C$ $\int \sin x dx = -\cos x + C$ $\int \sec^2 x dx = \tan x + C$

$\int \csc^2 x dx = -\cot x + C$ $\int \sec x \tan x dx = \sec x + C$ $\int \csc x \cot x dx = -\csc x + C$

$\int \frac{1}{x} dx = \ln|x| + C$ $\int \tan x dx = -\ln|\cos x| + C$ $\int \cot x dx = \ln|\sin x| + C$

$\int \sec x dx = \ln|\sec x + \tan x| + C$ $\int \csc x dx = -\ln|\csc x + \cot x| + C$ $\int e^x dx = e^x + C$

$\int a^x dx = \frac{a^x}{\ln a} + C$ $\int \frac{1}{\sqrt{a^2 - x^2}} dx = \arcsin\left(\frac{x}{a}\right) + C$

$\int \frac{1}{x^2 + a^2} dx = \frac{1}{a}\arctan\left(\frac{x}{a}\right) + C$ $\int \frac{1}{x\sqrt{x^2 - a^2}} dx = \frac{1}{a}\operatorname{arcsec}\left(\frac{|x|}{a}\right) + C$

Justifications for Reimann Sums:

Left-Riemann Sums:

The sum is an *overestimate* if the curve is decreasing.
The sum is an *underestimate* if the curve is increasing.

Right-Riemann Sums:

The sum is an *overestimate* if the curve is increasing.
The sum is an *underestimate* if the curve is decreasing.

First Fundamental Theorem of Calculus:

$\int_a^b f'(x)dx = f(b) - f(a)$ (Finds the signed area between a curve and the x-axis)

Second Fundamental Theorem of Calculus:

$\frac{d}{dx}\int_a^x f(t)dt = f(x)$ $\frac{d}{dx}\int_a^{g(x)} f(t)dt = f(g(x)) \bullet g'(x)$

Average Value of a Function:

$$f_{avg} = \frac{1}{b-a}\int_a^b f(x)dx$$

Exponential Growth and Decay:

"The rate of change of a quantity is directly proportional to that quantity"

Gives the differential equation: $\frac{dy}{dt} = ky$

Which can be solved to yield: $y = Ce^{kt}$

Particle Motion Formulas:

Velocity: $v(t) = s'(t)$

Acceleration: $a(t) = v'(t) = s''(t)$

Speed: speed $= |v(t)|$

Average Velocity: (given $s(t)$) $\frac{s(b)-s(a)}{b-a}$

(given $v(t)$) $\frac{1}{b-a}\int_a^b v(t)dt$

Average Acceleration: (given $v(t)$) $\frac{v(b)-v(a)}{b-a}$

(given $a(t)$) $\frac{1}{b-a}\int_a^b a(t)dt$

Displacement: $\int_a^b v(t)dt$

Total Distance: $\int_a^b |v(t)|dt$

Position at b: $s(b) = s(a) + \int_a^b v(t)dt$

Areas in a Plane:

Perpendicular to x-axis: $\int_a^b [f(x)-g(x)]dx$ $f(x)$ is top curve, $g(x)$ is bottom curve, a and b are x-coordinates of point of intersection

Perpendicular to y-axis: $\int_a^b [f(y)-g(y)]dy$ $f(y)$ is right curve, $g(y)$ is left curve, a and b are y-coordinates of point of intersection

Volumes Around a Horizontal Axis of Rotation or Perpendicular to x-axis:

Disc: $V=\int_a^b \pi r^2 dx$ a and b are x-coordinates

Washer: $V=\int_a^b \left[\pi R^2-\pi r^2\right] dx$ a and b are x-coordinates

Slab (Cross Section): $V=\int_a^b A(x)dx$ $A(x)$ is the area formula for the cross section

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Volumes Around a Vertical Axis of Rotation or Perpendicular to y-axis:

Disc: $V=\int_a^b \pi r^2 dy$ a and b are y-coordinates

Washer: $V=\int_a^b \left[\pi R^2-\pi r^2\right] dy$ a and b are y-coordinates

Slab (Cross Section): $V=\int_a^b A(y)dy$ $A(y)$ is the area formula for the cross section