

Proving the Quadratic Formula - 3/25/13 www.askmath.weebly.com

The Quadratic formula is a very useful equation. It tells us the x-intercepts of a quadratic equation without having to graph, or factor. It is a simple and frequently used formula in Algebra.

Let us start by the general form of a quadratic: $y = ax^2 + bx + c$

We then set this equal to zero because we want to find the x-intercepts also called "roots".

$$\begin{aligned}ax^2 + bx + c &= 0 \\ax^2 + bx &= -c \quad \text{Subtract } c \text{ on both sides} \\a\left(x^2 + \frac{b}{a}x\right) &= -c \quad \text{Factor out the } a \\a\left(x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2\right) &= a \cdot \left(\frac{b}{2a}\right)^2 - c \quad \text{Complete the square} \\ \left(x + \frac{b}{2a}\right)^2 &= \frac{a \cdot \left(\frac{b^2}{4a^2}\right) - c}{a} \quad \text{Divide by } a \\ \left(x + \frac{b}{2a}\right)^2 &= \frac{b^2}{4a^2} - \frac{c}{a} \quad \text{Join fractions} \\ x + \frac{b}{2a} &= \pm \sqrt{\frac{b^2}{4a^2} - \frac{4ac}{4a^2}} \quad \text{Take square root} \\ x &= \pm \sqrt{\frac{b^2}{4a^2} - \frac{4ac}{4a^2}} - \frac{b}{2a} \quad \text{Subtract } \frac{b}{2a} \\ x &= \pm \sqrt{\frac{b^2 - 4ac}{4a^2}} - \frac{b}{2a} \quad \text{Simplify fractions} \\ x &= \pm \frac{\sqrt{b^2 - 4ac}}{2a} - \frac{b}{2a} \\ x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\end{aligned}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$