Proving the vertex of a parabola to be equal to $x = \frac{-b}{2a}$

$f(x) = ax^2 + bx + c = 0$

The derivative shows us the slope of a functions as it passes an increment of h. As we get closer and closer to an x-value, then the instantaneous slope of the function can be very accurate. One way to do this is to take the limit of the functions as h goes closer to the x value. The derivative of a function is denoted by the "′" symbol.

Definition of a derivative: $\boxed{\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}}$

$$
\begin{aligned}
&= \lim_{h \to 0} \frac{a(x+h)^2 + b(x+h) + c - ax^2 - bx - c}{h} && (1) \\
&= \lim_{h \to 0} \frac{ax^2 + 2hxa + h^2a + bx + bh + c - ax^2 - bx - c}{h} && (2) \\
&= \lim_{h \to 0} \frac{2hxa + h^2a + bh}{h} && (3) \\
&= \lim_{h \to 0} \frac{h\,(2xa + ha + b)}{h} && (4) \\
&= \lim_{h \to 0} (2xa + ha + b) && (5) \\
& && (6)
\end{aligned}
$$

$\boxed{f'(x) = 2xa + b}$

At the vertex of a parabola, the instantaneous slope of the tangent line at the min/max is $y = 0$. The derivative of a function tells us the slope of the tangent line, so we set the derivative equal to zero and solve for x.

$$
\begin{aligned}
0 &= 2xa + b \\
-b &= 2xa
\end{aligned}
$$

$\boxed{x = \frac{-b}{2a}}$